

Painless causality in defect calculations

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Abstract

Topological defects must respect causality, a statement leading to restrictive constraints on the power spectrum of the total cosmological perturbations they induce. Causality constraints have for long been known to require the presence of an under-density in the surrounding matter compensating the defect network on large scales. This so-called compensation can never be neglected and significantly complicates calculations in defect scenarios, eg. computing cosmic microwave background fluctuations. A quick and dirty way to implement the compensation are the so-called compensation fudge factors. Here we derive the complete photon-baryon-CDM backreaction effects in defect scenarios. The fudge factor comes out as an algebraic identity and so we drop the negative qualifier “fudge”. The compensation scale is computed and physically interpreted. Secondary backreaction effects exist, and neglecting them constitutes the well-defined approximation scheme within which one should consider compensation factor calculations. We quantitatively assess the accuracy of this approximation, and conclude that the considerable pains associated with improving on it are often a waste of effort.

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I. INTRODUCTION

When a defect network is formed, causality and energy conservation demand that there must be a compensating under-density in the background matter and radiation. This compensation is exactly anti-correlated with the defects and is of a comparable intensity. It acts as a source for the gravitational potential which in turn drives radiation perturbations. For this reason the compensation cannot be ignored in CMB calculations. In previous work [1,2] the compensation has been included by use of compensation fudge factors. These ensure that the overall perturbations have a large-scale behaviour consistent with the causality constraints [3,4]. However their exact form was never justified by an analytical identity, and hence the words “fudge factor” qualifying them.

In this paper we show that by choosing a suitable gauge it is possible to derive analytically an expression for the compensation which contains a term in the form of the previously discussed fudge factor. Along with the compensation factor we find that there are also terms representing the backreaction from baryons, CDM and radiation which cannot be predicted *a priori*. We find however that these terms are sub-dominant compared to the defects for any reasonable values of the Hubble constant and the baryon fraction today. The physical implication is that it is the defect rather than any other component which dominates the spectrum of radiation perturbations. Hence by choosing a suitable defect it is possible to have a range of spectra which do not necessarily have the characteristic out of phase spectra associated with defect calculations, as shown in [2].

This paper is organised as follows. In Section II review the equations for a system of photons, baryons, CDM, and defects, in the tight-coupled approximation. We present a trick for easily considering a defect component within a multi-fluid formalism, such as the one presented in Kodama and Sasaki [5]. We show how the various gauge-invariant formulations correspond to nothing but different choices of full gauge fixing. We look into all popular choices of gauge. We argue in favour of the flat slicing gauge for the discussion of feedback mechanisms, the compensation, or the causality constraint. Then in Section III we present an algebraic manipulation which allows splitting compensation into two terms, one of which is purely a compensation fudge factor. We evaluate then, in Section IV, the quantitative impact of dropping the terms other than the compensation factor term. We find that one would need rather extreme values of the Hubble constant and baryon fraction for these extra terms to have much qualitative effect. In the concluding section we digress on the metaphysical implications of this results, and the practical application which we intend to give it.

II. THE BASIC EQUATIONS

We consider the epoch before recombination when it is a good approximation to treat all the cosmic components as a fluid. This is the so-called tight-coupled approximation, which we shall use to develop all our arguments. We will consider a scenario where the Universe is made up of radiation (corresponding variables labelled by γ), baryons (b), cold dark matter (c), and a defect component (s) where the baryons and radiation are tightly coupled (see eg. [8] for a quantitative definition).

Whenever we intend to show the generality of our results we shall also consider a set of extra generic components, denoted by α , which may be neutrinos or whatever personal taste requires. In order to extract intuition from our arguments we shall sometimes assume that only defects and radiation are relevant, a statement valid deep in the radiation dominated epoch. The actual calculations will however always be valid considering the other components. This is necessary for generality, since matter-radiation equality may (and in fact usually does) happen before recombination.

We use the gauge-invariant formalism in all the guises discussed in Kodama and Sasaki [5] (KS from now on). These different formulations, one should point out, correspond to different choices of gauge. A set of a priori gauge dependent variables defined in a fully fixed gauge is of course gauge invariant. The different possible gauge-invariant contrast variables correspond to nothing but the density contrast as measured in different fully fixed gauges. There is sometimes an inappropriate feeling that “gauge-invariant” and “gauge-dependent” methods are two separate tool boxes. They are in fact one and the same thing. The only exception to this statement is the synchronous gauge [6]. This choice of gauge leaves a residual gauge freedom and therefore variables defined in synchronous gauge can never be related algebraically to gauge invariant variables (see [7] for a good dictionary).

In this paper we adopt a multi-lingual attitude towards cosmic perturbation formalisms. We shall use a variety of density contrast variables Δ_α for a generic component α . These will be indexed in the following way. No extra index after the component index α denotes density contrast in the α -component rest frame (not in the total rest frame which we found a mess in the presence of defects). An index s denotes the density contrast in the Newtonian slicing (where the perturbed cosmological flow appears to have no shear). An index g denotes the density contrast in the flat slicing (where the equal time slices have no scalar curvature).

We consider all these different gauges in order to connect with previous work. However the main remark in this and the next section is that by choosing the flat slicing contrast variables two desirable (at least for defect practitioners) features may be achieved. Firstly, one gets rid of potential time derivatives in the equations for the radiation (and also for baryons and CDM, but this can also be achieved in other gauges). This is a major technical improvement on the formalism. It allows performing all calculations invoking only defect stress energy components and not their time derivatives. Structure functions for time derivatives are notoriously noisier in defect simulations.

Secondly, as shown in the next Section, in the flat slicing gauge the radiation backreaction naturally separates into two terms. The first is required by the Traschen integral constraints [3] and cannot be set to zero in any approximation scheme, otherwise causality is grossly violated. We will however find an exact expression for this term made up of a factor (independent of the perturbation variables) times a set of defect variables. This factor happens to have the same form as the “compensation fudge factor”. Furthermore the “compensation scale”, left undetermined in compensation factor calculations, can be computed. Hence fudging is an exact equality made obvious in the flat-slice gauge. The second backreaction term is truly unpredictable, but we will be able to show that it is not required by causality, and it is qualitatively sub-dominant.

A. A defect component in a multi-fluid formalism

We shall add to the multi-fluid formulation of Kodama and Sasaki a defect component. This can be best implemented by noticing that defects have no background stress-energy. We may then regard defects as a fluid for which the background energy and pressure are zero, the perturbation variables are infinite, but the product of the background and perturbation variables is finite and equals the defect stress-energy.

More mathematically let the defects stress-energy tensor $\Theta_{\mu\nu}$ be a pure scalar so that it may be written as

$$\begin{aligned}\Theta_{00} &= \rho^s \\ \Theta_{0i} &= k_i v^s \\ \Theta_{ij} &= p^s \delta_{ij} + (k_i k_j - \frac{1}{3} \delta_{ij} k^2) \Pi^s\end{aligned}\tag{1}$$

Then let us consider a component $\alpha = d$ with $\rho_d = p_d = 0$, infinite perturbation variables (eg. $\delta_d = \infty$), but finite products of the two. From the way perturbation variables are defined in KS [5] from the stress-energy tensor we can then write

$$\begin{aligned}a^2 \rho_d \delta_d &= \rho^s \\ a^2 (\rho_d + p_d) v_d &= k v^s \\ a^2 p_d \Pi_d^T &= k^2 \Pi^s \\ a^2 p_d \Pi_d^L &= p^s\end{aligned}\tag{2}$$

Because the background stress-energy of defects is zero, defect variables are gauge-invariant by themselves. However care must be taken when identifying KS defect variables with defect variables. For instance, it may happen that a gauge invariant density contrast variable is given by a combination of defect variables. Using Eqns. 2 we can find the identifications:

$$a^2 \rho_d \Delta_d = \rho^s + 3 h v^s\tag{3}$$

$$a^2 \rho_d \Delta_{sd} = a^2 \rho_d \Delta_{gd} = \rho^s\tag{4}$$

For all other variables there is no ambiguity, as the extra terms required to turn gauge-dependent variables into gauge independent ones simply vanish. For instance

$$a^2 (\rho_d + p_d) V_d = a^2 (\rho_d + p_d) (v_d - \dot{H}_T/k) = k v^s\tag{5}$$

$$a^2 p_d \Pi_d^L = p^s\tag{6}$$

$$a^2 p_d \Pi_d^T = k^2 \Pi^s\tag{7}$$

The conservation equations for the defect component may be written as;

$$\dot{\rho}^s + h(3p^s + \rho^s) + k^2 v^s = 0\tag{8}$$

$$\dot{v}^s + 2h v^s - p^s + \frac{2}{3} k^2 \Pi^s = 0.\tag{9}$$

which can be derived from the conservation equations in KS with the identifications made above. The gauge-invariant potentials Φ and Ψ can be obtained from the Einstein's equations

in KS. These are now sourced by a total density contrast and anisotropic stress which contains defects. We choose to separate the defects from all other components. Hence in all formulae in KS containing totals the following replacements should be introduced

$$a^2 \rho \Delta_T \rightarrow a^2 \rho \Delta_T + \rho^s + 3hv^s = a^2 \sum \rho_\alpha \Delta_\alpha + \rho^s + 3hv^s \quad (10)$$

$$a^2(p + \rho)V_T \rightarrow a^2(p + \rho)V_T + kv^s = a^2 \sum (p_\alpha + \rho_\alpha)V_\alpha + kv^s \quad (11)$$

$$a^2 p \Pi_T^T \rightarrow a^2 p \Pi_T^T + k^2 \Pi^s = a^2 \sum p_\alpha \Pi_\alpha^T + k^2 \Pi^s \quad (12)$$

Bearing this in mind, the Einstein equations in the presence of defects may now be read off from KS as

$$k^2 \Phi = 4\pi \left(a^2 \rho \Delta_T + \rho^s + 3hv^s \right) \quad (13)$$

$$\Phi + \Psi = -8\pi \left(a^2 \frac{p \Pi_T^T}{k^2} + \Pi^s \right) \quad (14)$$

In the scenario we are considering the total density contrast, putting defects aside, is given by

$$\rho \Delta_T = \rho_b \Delta_b + \rho_\gamma \Delta_\gamma + \rho_c \Delta_c \quad (15)$$

The fluids viscosity Π_T^T is entirely due to the photons brightness quadrupole [8]:

$$\Pi = \frac{12}{5} \Theta_2 \quad (16)$$

and can be set to zero in the tight-coupling limit.

B. The Newtonian slicing equations

The Newtonian slicing equations for the radiation are what leads to the Hu and Sugiyama (HS) formalism [8]. During tight-coupling the photon system is described in HS by the monopole and dipole components of the brightness function, Θ_0 and Θ_1 . In the fluid description this corresponds to the Newtonian slicing variables

$$\Theta_0 = \Delta_{s\gamma}/4 \quad (17)$$

$$\Theta_1 = V_\gamma \quad (18)$$

It can be checked that, with this identification, the conservation equations in KS for radiation become the HS equations:

$$\begin{aligned} \dot{\Theta}_0 &= -\frac{k}{3} \Theta_1 - \dot{\Phi} \\ \dot{\Theta}_1 &= -\frac{\dot{R}}{1+R} \Theta_1 + \frac{k}{1+R} \Theta_0 + k\Psi \end{aligned} \quad (19)$$

where $R = \frac{3\rho_b}{4\rho_\gamma}$ is the scale factor normalised to 3/4 at photon-baryon equality. It is for these equations that HS propose a WKB solution, which was used in the study of defect

Doppler peaks in [2]. If one uses this gauge for the radiation one must however change to the comoving gauge before computing the potentials Ψ and Φ . This can be easily done by means of

$$\Delta_\gamma = 4\left(\Theta_0 + h\frac{\Theta_1}{k}\right) \quad (20)$$

C. The comoving gauge

In Hu and White (HW) [9] the issue of backreaction is addressed in the comoving gauge (the word gauge being replaced by “representation”). A temperature variable is defined such that

$$T = \Delta_\gamma/4 = \Theta_0 + h\frac{\Theta_1}{k} \quad (21)$$

and an horrible set of equations is derived for them. The comoving gauge has the advantage that it is the natural gauge for representing the baryons, since in tight coupling baryons and photons share the same rest frame. As shown in HS and HW, the baryons’ density contrast Δ_b and velocity V_b satisfy the conditions

$$\dot{\Delta}_b = \frac{3}{4}\dot{\Delta}_\gamma \quad (22)$$

$$V_b = V_\gamma \quad (23)$$

which can be rewritten by defining the entropy as $s = \Delta_b - (3/4)\Delta_\gamma$, and rewriting the first equation as $\dot{s} = 0$.

The comoving gauge is also the gauge where the Traschen integral constraints are written [3,4]. In an expanding Universe a set of energy conservation laws apply to perturbation variables [3]. When applied to a Universe which is initially unperturbed, and then causally made inhomogeneous, these constraints translate into a stringent requirement on the large scale power spectrum of these perturbations [4]. This requirement is roughly that the power spectrum of the total energy perturbation in the comoving gauge goes like k^4 for small k . In the presence of defects the energy density subject to this law is:

$$\mathcal{U} = a^2\rho\Delta_T + \rho^s + 3hv^s \quad (24)$$

which is also the source of the gauge-invariant potential Φ . Hence one can rephrase the causal constraint as the requirement that Φ goes to a constant at low k (white-noise).

The lack of superhorizon correlations in the defect network requires the power spectrum $P(\rho^s)$ to have a white noise low k tail. Energy conservation Eqn. (8) requires that v^s also have a white noise low k tail. Hence the causal constraint entails the need for the compensation: a low k white-noise tail in the power spectrum of non-defect matter. The compensation must be exactly anticorrelated with the defects’ tail. The quantity to be cancelled is $\rho^s + 3hv^s$, and not just ρ^s . The density forced to have a k^4 power spectrum is \mathcal{U} and not just a combination of Δ_T and ρ^s .

D. The flat-slicing gauge

We can also define a temperature perturbation in the flat-slicing gauge [16]:

$$\Delta_0 = \Theta_0 + \Phi = \Delta_{g\gamma}/4 \quad (25)$$

In this gauge the photon equations are:

$$\begin{aligned} \dot{\Delta}_0 &= -\frac{k}{3}\Theta_1 \\ \dot{\Theta}_1 &= -\frac{\dot{R}}{1+R}\Theta_1 + \frac{k}{1+R}(\Delta_0 - \Phi) + k\Psi \end{aligned} \quad (26)$$

As announced before one gets rid of the potential time derivatives in this gauge. We will also find it useful to represent CDM in this gauge, so that CDM equations are:

$$\begin{aligned} \dot{\Delta}_{gc} &= -kV_c \\ \dot{V}_c &= -hV_c + k\Psi \end{aligned} \quad (27)$$

There is a good mathematical reason why this gauge may be better for discussing causality and compensation issues. This is the gauge where the fluid equations of motion more resemble Minkowski space-time equations of motion. Hence energy variables in this gauge are akin to the pseudo-energy usually defined in the synchronous gauge, and used to introduce the compensation [14,15].

III. EXACT COMPENSATION FACTORS

We start with an algebraic remark. The source for the Einsteins equations are energy density variables in the comoving gauge. In the scenario we are considering

$$k^2\Phi = 4\pi\{a^2\rho(\Omega_b\Delta_b + \Omega_\gamma\Delta_\gamma + \Omega_c\Delta_c + \Omega_\alpha\Delta_\alpha) + \rho^s + 3hv^s\} \quad (28)$$

$$\Phi + \Psi = -8\pi\Pi^s \quad (29)$$

where α represents any other component we may have forgotten, with a sum over α implied, if need be. Now let us express baryons in terms of photons by means of Eq. (22) written as

$$\Delta_b = \frac{3}{4}\Delta_\gamma + s \quad (30)$$

and write all other sources in the flat-slice gauge:

$$\Delta_\gamma = 4\left(\Delta_0 - \Phi + h\frac{\Theta_1}{k}\right) \quad (31)$$

$$\Delta_c = \Delta_{gc} + 3\left(h\frac{V_c}{k} - \Phi\right) \quad (32)$$

$$\Delta_\alpha = \Delta_{g\alpha} + 3(1+w_\alpha)\left(h\frac{V_\alpha}{k} - \Phi\right) \quad (33)$$

With these rearrangements, the first Einstein equation becomes

$$k^2\Phi = 4\pi \left(a^2\rho \left(4\Omega_\gamma(1+R)\left(\Delta_0 - \Phi + h\frac{\Theta_1}{k} \right) + \Omega_b s + \Omega_c \left(\Delta_{cg} - 3\Phi + 3\frac{V_c}{k} \right) + \Omega_\alpha \left(\Delta_{\alpha g} + 3(1+w_\alpha) \left(-\Phi + \frac{V_\alpha}{k} \right) \right) \right) + \rho^s + 3hv^s \right) \quad (34)$$

The source term can now be split into 3 components:

$$k^2\Phi = S + S_1 + S_2 \quad (35)$$

where

$$S = 4\pi(a^2\rho_b s + \rho^s + 3hv^s) \quad (36)$$

$$S_1 = -4\pi a^2\rho(\Omega_\gamma(1+R) + 3\Omega_c + 3(1+w_\alpha)\Omega_\alpha)\Phi \quad (37)$$

$$S_2 = 4\pi a^2\rho(4\Omega_\gamma(1+R)(\Delta_0 + h\Theta_1/k) + \Omega_c(\Delta_{gc} + 3hV_c/k) + \Omega_\alpha(\Delta_{g\alpha} + 3hV_\alpha/k)) \quad (38)$$

S is made up of sources which drive the radiation-baryon-CDM system but which are external to them. We call it the external source. This may be a topological defect. An entropy perturbation may also be regarded as external since it evolves independently of all other perturbations (according to $\dot{s} = 0$). There is a fundamental difference between defects and entropy perturbations. Entropy perturbations satisfy $\dot{s} = 0$. Defect sources, on the contrary, satisfy Eqns. (8) and (9).

The photon-baryon-CDM system is also driven by a backreacting term, here split as $S_1 + S_2$. This reflects the fact that baryons, photons and CDM are driven by a potential which they are a source of. There is therefore a (linear) feedback effect which jeopardises for instance the use of the WKB solution in HS for defects. One could hope that defects are the main driving force, and try to neglect backreaction. However this back-reacting term incorporates the compensation. Setting $S_1 + S_2$ to zero is therefore an approximation which can never make sense, as it would imply a gross violation of the causality constraint. The potential Φ power spectrum would diverge like $1/k^4$ at small k rather than go to white noise.

However, in the flat-slice gauge we have an algebraic bootstrap which one may hope already reflects most of the physical feedback mechanism. This bootstrap is created by the term S_1 . Let us first set $S_2 = 0$. Then all the backreaction is predictable and fully determined by the defect sources. S_1 may be passed to the left hand side of the Einstein equation (35) and be incorporated in an equation where the external sources are simply multiplied by a factor independent of photon, baryon or CDM variables. More precisely

$$k^2\Phi = 4\pi\gamma_c(a^2\rho_b s + \rho^s + 3hv^s) \quad (39)$$

where

$$\gamma_c = \frac{1}{1 + (\chi_c/x)^2} \quad (40)$$

$$\chi_c^2 = \frac{3}{2}(h\eta)^2(4\Omega_\gamma(1+R) + 3\Omega_c + 3(1+w_\alpha)\Omega_\alpha) \quad (41)$$

The backreaction encoded in S_1 is not an independent feedback mechanism operating in the fluid and imprinting a fixed signature in the photons' power spectrum for any defect

theory. From this term we can never expect to derive an out-of-phase signature as the one attributed to defects in [9]. The backreaction contained in this term is fully driven by the defects alone, and can be made to behave in whatever way we want by properly designing the defect. It is not surprising that by considering only this backreaction effect we can place the primary Doppler peak anywhere, including the adiabatic and out-of-phase positions [2]. On the other hand by considering this term one is already taking into account the causality constraint. The potential Φ according to the new equation (39) already goes to white noise at small k .

If $S_2 \neq 0$ then one must add to equation (39) an extra source term, so that

$$k^2\Phi = \gamma_c(S + S_2) \quad (42)$$

This is an exact expression. The compensation factor has appeared as a result of algebra and the compensation scale is a well defined quantity dependent only on the unperturbed cosmological expansion dynamics. The compensation factor approximation is now the claim that we can set to zero S_2 . This is a well defined statement which we should be able to assess quantitatively. The term in S_2 is the truly unpredictable backreaction. The $S_1 + S_2$ split has allowed us to separate what is truly a problem and what is not. By doing so we have implemented an approximation scheme ($S_2 \approx 0$) where we may avoid the feedback problem without immediately doing something stupid, like violating the causality constraint.

Physically what this algebraic manipulation amounts to is the realization, made obvious in the flat-slice gauge, that the compensation is made up of 3 terms. One is the energy perturbation as it appears in a gauge where equal time surfaces appear to have no curvature. The other two are a potential perturbation describing the curvature of these slices, and a velocity term. The potential term is caused by the defects as well, and by dropping all other contributions we obtain a non pathological approximation scheme where the compensation is fully gravitational and perfectly correlated to the defect network.

The compensation scale χ_c varies from $\sqrt{6}$ in the radiation epoch to $\sqrt{18}$ in the matter epoch. The compensation scale depends purely on the expansion kinematics and is affected by the matter radiation transition. It can be written as

$$\chi_c^2 = 3\eta^2(h^2 - \dot{h}) = 12\pi a^2(p + \rho)\eta^2 \quad (43)$$

and if $h = \alpha/\eta$ then $\chi_c^2 = 3\alpha(\alpha + 1)$.

IV. QUANTITATIVE ARGUMENT IN FAVOUR OF COMPENSATION FACTORS

The question remains of how good an approximation setting $S_2 = 0$ is. We address this question quantitatively. For definiteness we use the source defined in [10]:

$$p^s = \frac{1}{\eta^{1/2}} \frac{\sin Ak\eta}{Ak\eta} \quad (44)$$

$$\Pi^s = 0 \quad (45)$$

bearing in mind that this ansatz may preclude arbitrary shifts in peaks' positions. This property is far from general, as shown in [11,12]. This issue is beyond the scope of this

paper, but will be addressed in a future publication [17]. We then solve equations (26) for radiation, (27) for CDM, (8) and (9) for the remaining defect variables, and (13) and (14) for the potentials (with (13) rewritten as in (42)). The baryons are solved implicitly by the tight-coupled conditions (22). By including S_2 as a source of the potential Φ in (42) one is considering the full backreaction effects, due to baryons, radiation, and CDM. One thus obtains the exact solution to this problem. By dropping the term in Δ_{gc} and V_c in S_2 one neglects the effects of CDM fluctuations on the CMB fluctuations. By setting $S_2 = 0$ one neglects the effects of backreaction altogether. In the last approximation the source is truly external, and is compensated purely by defect gravitational effects. This is the compensation factor approximation.

We have solved this problem for various values of the Hubble constant and baryon content of the Universe. These are parameterised by h , so that the Hubble constant nowadays is $H_0 = 100h\text{Kms}^{-1}\text{Mpc}^{-1}$, and $\Omega_b = \rho_{b0}/\rho_0$, the baryons density fraction nowadays. We expressed our results in terms of the effective temperature $\Theta_0 + \Psi = \Delta_0 + (\Psi - \Phi)$ and the dipole Θ_1 at last scattering $\eta = \eta_*$. This is because these are the quantities which are then projected onto C_l 's by free-streaming after the last scattering surface [8]. We have solved the full problem and compared the full answer with the effect of dropping CDM, and dropping all CDM-photon-baryon backreaction. In all plots these 3 calculation schemes will be represented by dash, dots, and lines, respectively.

The results for a source with $A = 1$ are plotted in Figure IV, and we now comment on them. We have chosen extreme values for Ω_b and h in order to emphasise the point we wish to make. For popular values of Ω_b and h the compensation factor approximation works very well. Clearly one needs rather high values for both h and Ω_b for the compensation factor approximation to become gross.

As h increases the time between equality and last scattering increases. As a result CDM fluctuations have time to start to grow while they still can interfere with the tightly coupled radiation. As a result the CDM contribution cannot really be neglected in scenarios with a large Hubble constant.

As Ω_b increases the so-called acoustic signature may be imparted on the peaks. This is an asymmetry in amplitude between odd and even peaks resulting from baryons shifting the zero level of the oscillations. After squaring, the peaks will appear alternately big and small. It is interesting to note two things. First the acoustic signature appears already in the compensation factor approximation, although less pronounced. Secondly, if one is to pay attention to detail, then CDM is as important as the baryons in imprinting the full acoustic signature.

Another feature present in the spectra is the shoulder preceding the peaks, ubiquitous in defect scenario. This is not present in Δ_0 and is due to the gravitational redshift term $\Psi - \Phi$. At large scales $\Delta_0 \approx 0$ and then goes negative. The potential term $\Psi - \Phi$ is white noise and positive as $k \rightarrow 0$, then goes to zero. This induces a pre-peak which is not acoustic, but merely a gravitational redshift effect at last scattering.

All in all backreaction seems to be negligible in the qualitative study of defects. Historically it has been known that backreaction in defect theories can never be neglected, as a result of the causality constraints. However we have now shown that backreaction can be exactly split into two terms. One is purely gravitational, makes sure that the causality constraints are satisfied, and can be predicted a priori from the defects by means of

a compensation factor. The other cannot be predicted a priori but it is not required by causality. It merely reflects the baryons, photons, and CDM trying to make a nuisance of themselves, rebelling against the driving force of the defects, trying to imprint a signature they are allowed to imprint whenever no driving force is present. However for all reasonable values of Ω_b and h this secondary backreaction is quantitatively sub-dominant. When driven by defects the radiation/baryon/CDM feedback effect is normally weaker than the external force they are subject to. Therefore, it turns out that in defect scenarios, once compensation factors are taken into account, backreaction is precisely something which can be neglected, certainly in any qualitative discussion of defect perturbations.

V. WASTE NOT - A DEFECT FACTORY

In this paper we stressed the technical difficulties induced when considering compensation in defect calculations but have shown how they can be effectively bypassed by means of compensation factors. The compensation results from a feedback mechanism which operates in the photon, CDM, baryon system when an external driving force is applied to them. Some of this backreaction is unpredictable in the sense that by specifying the driving force one does not specify the backreaction before the system of differential equations describing the whole system is evolved. However the dominant compensation term is always predictable in the sense that all one needs to do is to multiply some appropriate combination of defect stress energy components by a compensation factor.

The practical implication of this result is of course not to simplify the numerics of evolving the tight coupled equations; this problem is straightforward enough for simplification to be necessary. However there is another, more metaphysical, side to compensation considerations in the literature: this is the belief that backreaction effects force the Doppler peaks to be out of phase. This could only be a theorem if the compensation was the result of a self-regulatory feedback mechanism operating in the fluid, regardless of the details of the force driving it. The compensation described by the compensation factor, which we showed to be dominant, is precisely the opposite of this. It represents a feedback effect which is directly connected to the driving force, and which has a scale which can be manipulated by tuning the scale of the driving force. It was in fact shown in [2] that in the compensation factor approximation, a causal source could be designed so as to shift the Doppler peaks about, placing them on the adiabatic position or anywhere to its right.

The fact that the compensation factor approximation also seems to reproduce other effects, such as the acoustic signature, then provides us with the practical application of this work. If the compensation factor approximation works one can write down a solution expressing the monopole and dipole in the photons at last scattering as a function of a defect structure function. We can then invert this expression and instead write down a defect structure function which could give us a given monopole and dipole at last scattering. This trick can then be converted into a defect factory, allowing intelligent guesses of causal sources which exhibit effects under study. In particular one may use this defect factory to produce confusing defects: causal sources exhibiting allegedly inflationary signatures, such as the acoustic signature. Naturally once an educated guess has been made, one should then go and evolve the full system of tightly coupled equations, or even better solve a Boltzmann code for the proposed source. However there is no reason why one should go the complicated

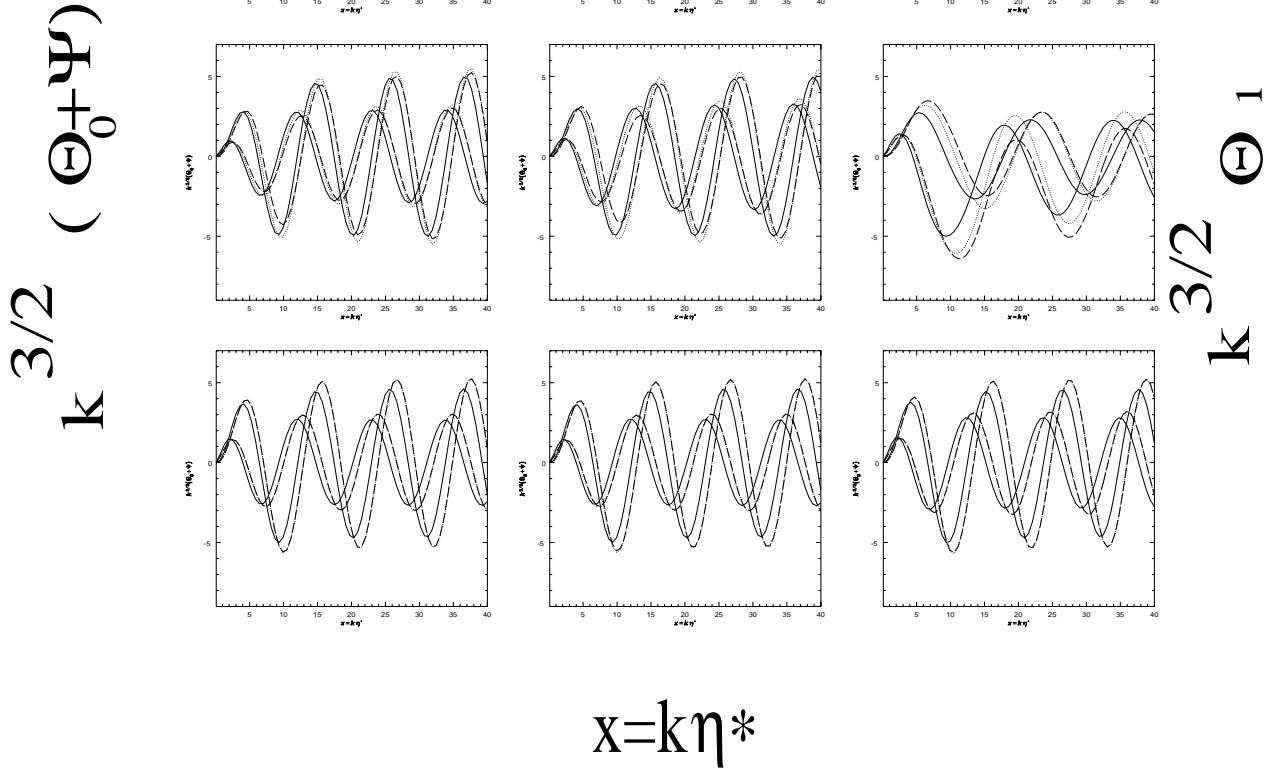


FIG. 1. We plot $k^{3/2}(\Theta_0 + \Psi)$ (curves which go negative first) and $k^{3/2}\Theta_1$ with the following conventions: the compensation factor approximation in lines, photon-baryon backreaction included in dots, all backreaction included in dash. From left to right $\Omega_b = 0.005, 0.05, 0.5$. From bottom to top $h = 0.1, 0.5, 0.8$.

way when guessing sources. In a future publication we shall use this approach in the study of confusing defects [17].

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